25[7].—WILLARD MILLER, JR., *Lie Theory and Special Functions*, Academic Press, Inc., New York, 1968, xv + 338 pp., 23 cm. Price $16.50.

An excellent description of this volume, its purpose as well as its scope, is given in the preface:

"This monograph is the result of an attempt to understand the role played by special function theory in the formalism of mathematical physics. It demonstrates explicitly that special functions which arise in the study of mathematical models of physical phenomena and the identities which these functions obey are in many cases dictated by symmetry groups admitted by the models. In particular it will be shown that the factorization method, a powerful tool for computing eigenvalues and recurrence relations for solutions of second order ordinary differential equations (Infeld and Hull [1]), is equivalent to the representation theory of four local Lie groups. A detailed study of these four groups and their Lie algebras leads to a unified treatment of a significant proportion of special function theory, especially that part of the theory which is most useful in mathematical physics.

"Most of the identities for special functions derived in this book are known in one form or another. Our principal aim is not to derive new results but rather to provide insight into the structure of special function theory. Thus, all of the identities obtained here will be given an explicit group-theoretic interpretation instead of being considered merely as the result of some formal manipulation of infinite series.

"The primary tools needed to deduce our results are multiplier representations of local Lie groups and representations of Lie algebras by generalized Lie derivatives. These concepts are introduced in Chapter 1 along with a brief survey of classical Lie theory. In Chapter 2 we state our main theme: Special functions occur as matrix elements and basis vectors corresponding to multiplier representations of local Lie groups. Chapters 3–6 are devoted to the explicit analysis of the special function theory of the complex local Lie groups with four-dimensional Lie algebras $\mathcal{G}(0, 0)$, $\mathcal{G}(0, 1)$, $\mathcal{G}(1, 0)$ and six-dimensional Lie algebra $\mathfrak{I}_6$. Many fundamental properties of hypergeometric, confluent hypergeometric, and Bessel functions are obtained from this analysis. Furthermore, in Chapter 7 it is shown that the representation theory of these four Lie groups is completely equivalent to the factorization method of Infeld and Hull.

"In Chapter 8 we determine the scope of our analysis by constructing the rudiments of a classification theory of generalized Lie derivatives. This classification theory enables us to decide in what sense the results of Chapters 3–6 are complete. Chapter 8 is more difficult than the rest of the book since it presupposes a knowledge of some rather deep results in local Lie theory and an acquaintance with the cohomology theory of Lie algebras. Hence, it may be omitted on a first reading.

"Finally, in Chapter 9 we apply some of the results of the classification theory to obtain identities for special functions which are not related to the factorization method.

"It should be noted that we will be primarily concerned with the representation theory of local Lie groups, a subject which was developed in the nineteenth century. The more recent and sophisticated theory of global Lie groups is by itself too narrow to obtain many fundamental identities for special functions. Also, unitary representations of Lie groups will occur only as special cases of our results; they will not be our primary concern.

"The scope of this book is modest: we study no Lie algebras with dimension greater than 6. Furthermore, in the six-dimensional case, $\mathfrak{G}_6$, we do not give complete results. (The so-called addition theorems of Gegenbauer type for Bessel functions would be obtained from such an analysis.) However, it should be clear to the reader that our methods can be generalized to higher dimensional Lie algebras.

"We will almost exclusively be concerned with the derivation of recursion relations and addition theorems. The manifold applications of group theory in the derivation of orthogonality relations and integral transforms of special functions will rarely be considered. For these applications see the encyclopedic work of Vilenkin. The overlap in subject matter between that book and this one is relatively small, except in the study of unitary representations of real Lie groups."

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26[7].—RUDOLF ONDREJKA, *Exact Values of* 2^n, $n = 1(1)4000$, ms. of 519 computer sheets deposited in the UMT file.

This impressive table of the exact values of the first 4000 powers of 2, which was computed in 1961 on an IBM 709 system, constitutes the first volume of an extensive unpublished series of such tables.

In private correspondence with the editors the author has revealed that in October and November 1966 and in the period from May through August 1967 he extended this initial table by computing the next 29,219 powers of 2 on an IBM 7090 system. These additional powers occupy a total of 27,023 computer sheets, arranged in 74 volumes, which are in the possession of the author.

A further statistic supplied by the author is that the total number of digits in all 75 volumes is 166,115,268. This digit count is also given for each volume; thus, the volume under review, for example, contains 2,410,843 digits. A useful index has been supplied for the entire set of tables.

The selection of 33,219 as the total number of entries in this immense tabulation was based on the author's plan to include all powers of 2 whose individual lengths do not exceed 10,000 decimal digits.

The tabular entries are clearly printed in groups of five digits, with 19 such pentads on each line. The appropriate exponent is printed beside each entry, and successive powers are conveniently separated by a double space.

In a brief abstract the author has added the information that tabular values were spot checked by comparison with the published results of others, particularly those relating to the known Mersenne primes.

J. W. W.

27[7].—H. R. AGGARWAL & VAHE SAGHERIAN, *An Extension of the Tables of the Quotient Functions of the Third Kind*, Stanford Research Institute, Menlo Park, Calif., ms. of 18 typewritten pages deposited in the UMT file.